MATHEMATICS **10. *TRIANGLES*** ***STD – IX***

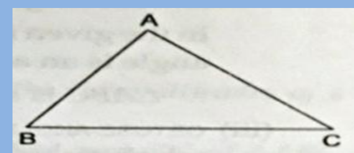
General instructions for Students: Whatever be the notes provided, everything must be copied in the Maths copy and then do the HOMEWORK in the same copy.

TRIANGLE *– A plane figure bounded by three line segments is called triangle.*

Three vertices $A,\ B$ and C

Three sides $AB,\ BC$ and CA

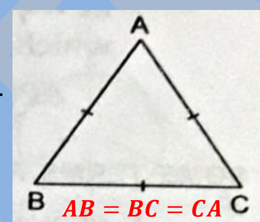
Three angles $\angle A,\ \angle B$ and $\angle C$



TYPES OF TRIANGLES ON THE BASIS OF SIDES

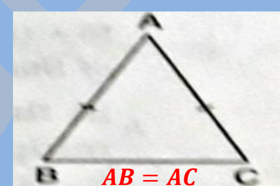
1. *EQUILATERAL TRIANGLE* –

A triangle having all sides equal is called an equilateral triangle.

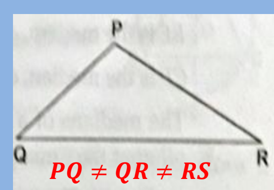


2. *ISOSCELES TRIANGLE* –

A triangle having two sides equal is called an isosceles triangle.



3. *SCALENE TRIANGLE* – *If all the sides of a triangle are unequal, it is called a scalene triangle.*

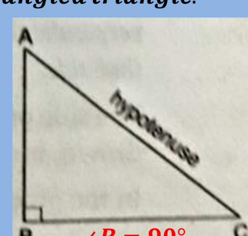


PERIMETER OF A TRIANGLE - ***The sum of the lengths of three sides of a triangle is called its perimeter.***

TYPES OF TRIANGLE ON THE BASIS OF ANGLES

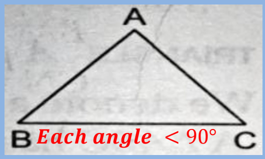
1. *RIGHT – ANGLED TRIANGLE* : –

If one angle of a triangle is right angle $(= 90°)$, *it is called right – angled triangle.*



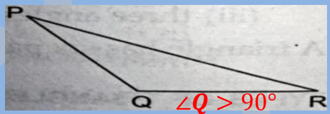
2. ACUTE – ANGLED TRIANGLE : –

If all the three angles of a triangle are acute (less than 90°), it is called an acute – angled triangle.



3. OBTUSE – ANGLED TRIANGLE : –

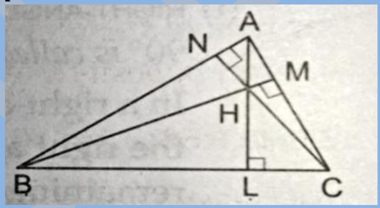
If one angle of a triangle is obtuse (greater than 90°), it is called obtuse – angled triangle.



SOME TERMS CONNECTED WITH A TRIANGLE

ALTITUDE – *Perpendicular from a vertex of a triangle to the opposite side is called an altitude.*

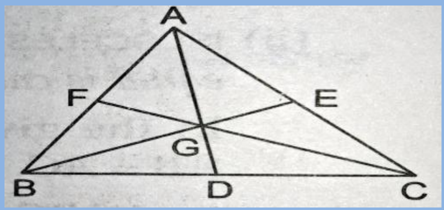
A triangle has three altitudes.



ORTHOCENTRE – *The point of intersection of all three altitudes of a triangle is called its orthocentre.*

MEDIANS – *The straight line joining a vertex of a triangle to the mid – point of the opposite side is called a median of the triangle.*

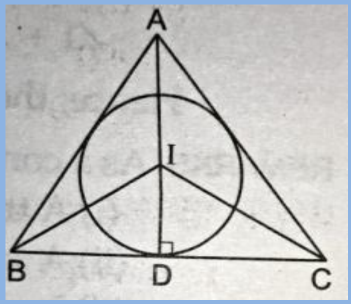
A triangle has three medians.



CENTROID – *The point of intersection of all three medians of a triangle is called its centroid.*

The centroid of a triangle divides every median in the ratio of 2 : 1

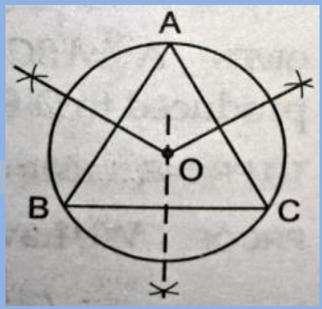
$\therefore$ *G is the centroid of $\triangle ABC$*



INCENTRE AND INCIRCLE

The point of intersection of the internal bisectors of the angles of a triangle is called its incentre.

Moreover, incentre is the centre of the circle which touches all sides of $\triangle$ ABC and this circle is called incircle of $\triangle$ ABC.



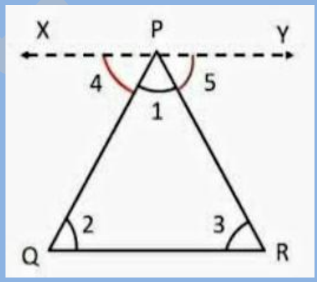
CIRCUMCENTRE AND CIRCUMCIRCLE

The point of intersection of the perpendicular bisectors of the sides of a triangle is called its circumcentre.

Moreover, circumcentre is the centre of the circle which passes through the vertices of $\triangle$ ABC and this circle is called circumcircle of $\triangle$ ABC.

ANGLES SUM PROPERTY OF A TRIANGLE

Statement : ***The sum of the angles of a triangle is 180°***



Given: ***A $\triangle$ PQR.***

To prove : $\angle 1 + \angle 2 + \angle 3 = 180°$

Construction: ***Through P, draw a line XY $\parallel$ QR***

Proof: ***XY $\parallel$ QR and PQ is a transversal***

$\therefore\ \angle 2 = \angle 4$ …………………………..(i) [***Alt. Int.*** $\angle s$]

XY $\parallel$ QR and PR is a transversal

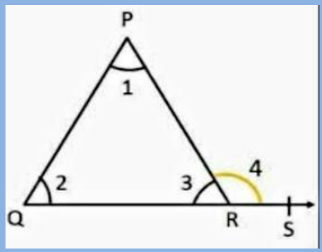
$\therefore\ \angle 3 = \angle 5$ …………………………..(ii) [***Alt. Int.*** $\angle s$]

Now, $\angle 1 + \angle 4 + \angle 5 = 180°$ [$\because$ ***XPY is a straight line***]

$\angle 1 + \angle 2 + \angle 3 = 180°$ ***By (i) and (ii)*** ***Proved.***

AN EXTERIOR ANGLE PROPERTY OF A TRIANGLE

Statement : ***An exterior angle of a triangle is equal to sum of its interior opposite angles***



Given: ***A $\triangle$ PQR whose side QR produced to S.***

To prove : $\angle 4 = \angle 1 + \angle 2$

Proof: ***We know that*** $\angle 1 + \angle 2 + \angle 3 = 180°$ ……..(i)

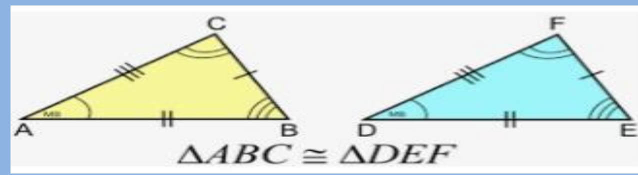
[***By Angles sum property of a triangle***]

Also, $\angle 3 + \angle 4 = 180°$ ……………..(ii) [***linear pair***]

$\therefore\ \angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$ [***By (i) and (ii)***]

Hence, $\angle 1 + \angle 2 = \angle 4$ ***Proved.***

CONGRUENCE OF TRIANGLES



If $\triangle ABC \cong \triangle DEF$, *then*

(i) ***their corresponding sides are equal*** *i.e.* $AB = DE,\ BC = EF$ **and** $AC = DF$

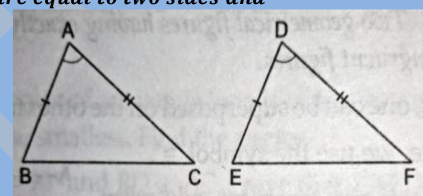
(ii) ***and their corresponding angles are equal*** *i.e.* $\angle A = \angle D,\ \angle B = \angle E$ **and** $\angle C = \angle F$.

We shall use the abbreviation " C. P. C. T." *for* " *Corresponding Parts of Congruent Triangles*"

CRITERIA FOR CONGRUENCE OF TRIANGLES

SAS congruence rule (Axiom)

Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle.



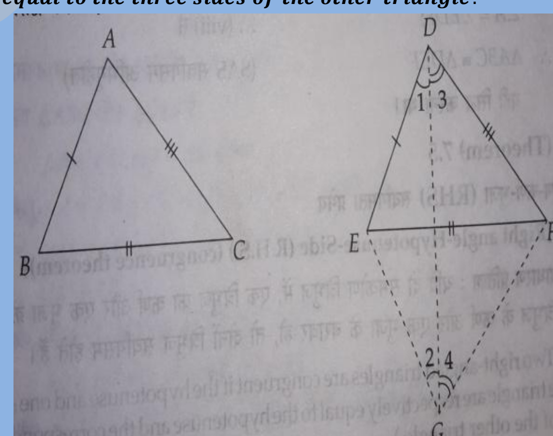
In $\triangle ABC$ *and* $\triangle DEF$, $AB = DE,\ \angle BAC = \angle EDF$ *and* $AC = DF$

$\therefore\ \triangle ABC \cong \triangle DEF$

- ***SSS congruence rule*** (*All three corresponding sides are equal*)
- ***ASA congruence rule*** (*Two angles and included side*)
- ***AAS congruence rule*** (*Two angles and one side*)
- ***RHS congruence rule*** (*In two right triangles hypotenuse and one side*)

SSS congruence rule :

Statement: *Two triangles are congruent if the three sides of a triangle are equal to the three sides of the other triangle.*



Given: *In* $\triangle ABC$ *and* $\triangle DEF$, $AB = DE,\ BC = EF$ *and* $AC = DF$

To prove: $\triangle ABC \cong \triangle DEF$

Construction: *Let BC is the longest side, Draw EG such that,* $\angle FEG = \angle ABC$, $EG = AB$. *We join GF and GD.*

Proof: *In* $\triangle ABC$ *and* $\triangle GEF$,

$AB = EG$ (*By Construction*)

$\angle ABC = \angle FEG$ (*By Construction*)

$BC = EF$ (*Given*)

$\therefore\ \triangle ABC \cong \triangle GEF$ (*By SAS Congruence Rule*)

$\angle A = \angle G$ *and* $AC = GF$ ……………… (i) (*By C.P.C.T.*)

Now, $AB = EG$ (*By Construction*)

$AB = DE$ (*Given*)

$\therefore\ EG = DE$ …………… (ii)

Similarly, $GF = DF$ ……….…..(iii)

In $\triangle DEG$, $DE = EG$ *By* (ii) ($\angle$*s opposite to equal sides are equal*)

$\therefore \quad \angle 1 = \angle 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(iv)$

Similarly, $\quad$ *In* $\triangle DFG, \quad \angle 3 = \angle 4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(v) \quad (\angle s$ *opposite to equal sides are equal*$)$

$(iv) + (v) \quad \Rightarrow \quad \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle EDF = \angle EGF \quad \ldots\ldots\ldots\ldots\ldots.(vi)$

But $\quad \angle A = \angle EGF \quad \therefore \ \angle A = \angle EDF \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(vii)$

Now, $\quad$ *In* $\triangle ABC$ *and* $\triangle DEF$

$AB = DE \quad (Given)$

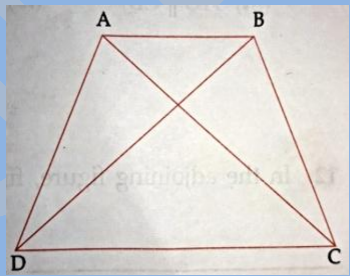
$AC = DF \quad (Given)$

$\angle A = \angle EDF \quad By\ (vii)$

$\therefore \quad \triangle ABC \cong \triangle DEF \quad (By\ SAS\ Congruence\ Rule)$ **Proved.**

EXERCISE – 10.1

Q.NO. 5. In the adj. figure, $AD = BC$ and $BD = AC$. Prove that $\angle ADB = \angle BCA$ and $\angle DAB = \angle CBA$.



Solution: $\quad$ *Given* $\quad AD = BC$ *and* $BD = AC$

In $\triangle ADB$ *and* $\triangle BCA$

$AD = BC \quad (Given)$

$BD = AC \quad (Given)$

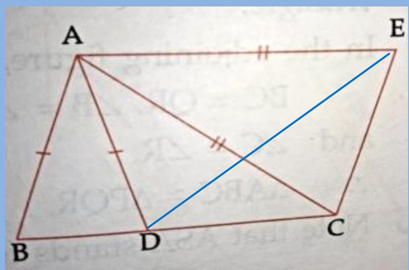
$AB = AB \quad (common)$

$\therefore \ \triangle ADB \cong \triangle BCA \quad (SSS\ Congruence\ Rule)$

$\therefore \ \angle ADB = \angle BCA \quad (By\ \ C.P.C.T.)$

And $\ \angle DAB = \angle CBA \quad (By\ \ C.P.C.T.)$ **Proved.**

Q.NO. 8. In the adj. figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle CAE$. Show that $BC = DE$.



Given: $\quad AC = AE, \quad AB = AD \quad$ *and* $\ \angle BAD = \angle CAE$

Construction: $\quad$ *We join DE.*

To show: $\quad BC = DE$

Proof: $\quad$ *In* $\triangle ABC$ *and* $\triangle ADE$

$AB = AD \quad (Given)$

$AC = AE \quad (Given)$

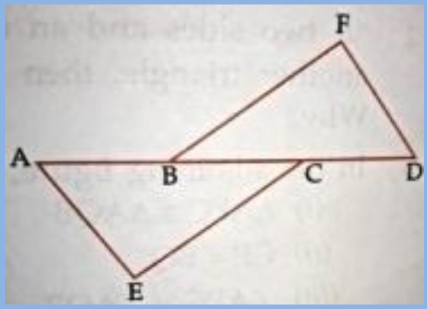
$\angle BAD = \angle CAE \quad (Given)$

$\angle BAD + \angle DAC = \angle CAE + \angle DAC \quad [\,Adding\ both\ sides\ \ \angle DAC\,]$

$\angle BAC = \angle DAE$

$\therefore \quad \triangle ABC \cong \triangle ADE \quad (SAS\ Congruence\ Rule) \quad \therefore \quad BC = DE \quad (By\ \ C.P.C.T.)$ **Proved.**

$Q.NO.\ 9.$ ***In the adj. figure,*** $AB = CD$, $CE = BF$ ***and*** $\angle ACE = \angle DBF$.



Prove that (i) $\triangle ACE \cong \triangle DBF$ (ii) $AE = DF$.

Given $AB = CD$, $CE = BF$ ***and*** $\angle ACE = \angle DBF$

To prove: (i) $\triangle ACE \cong \triangle DBF$ (ii) $AE = DF$.

Proof: ***In*** $\triangle ACE$ ***and*** $\triangle DBF$

$CE = BF$ (***Given***)

$\angle ACE = \angle DBF$ (***Given***)

$AB = CD$ (***Given***)

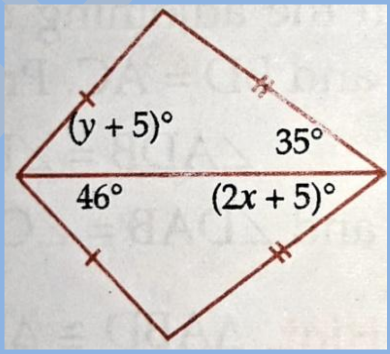
$AB + BC = CD + BC$ [***Adding both sides*** BC]

$AC = BD$

$\therefore$ $\triangle ACE \cong \triangle DBF$ (***SAS Congruence Rule***)

$\therefore$ $AE = DF$ (***By C.P.C.T.***) ***Proved.***

$Q.NO.\ 12.$ ***In the adj. figure, find the value of*** x ***and*** y.



Solution: ***Two given triangles are congruent by SSS CONGRUENCY RULE***

their coressponding angles are equal.

$\therefore$ $(y + 5)^\circ = 46°$

$\Rightarrow$ $y = 46 - 5$

$\Rightarrow$ $y = 41$ ***Ans.***

Now, $(2x + 5)^\circ = 35°$

$\Rightarrow$ $2x = 35 - 5$

$\Rightarrow$ $2x = 30$

$\Rightarrow$ $x = 15$ ***Ans.***

HOMEWORK

EXERCISE $- 10.1$

QUESTION NUMBERS: 4, 6, 7 ***and*** 11.